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ABSTRACT

Linear regression examines the relationship between one or more independent (predictor) variables and a dependent variable. By using a particular formula, regression determines the weights needed to minimize the error term for a given set of predictors. With one predictor variable, the relationship between the predictor and the dependent variable is linear. With two predictors, this relationship becomes planar, and with three or more predictors, this relationship becomes hyper planar. By examining three-dimensional representations of the data, a researcher can gain greater insight into the data. The recent report of the American Psychological Association Task Force on Statistical Inference, published in the August 1999 issue of the "American Psychologist," emphasizes the value and importance of using graphics to understand and communicate data dynamics. (Contains 11 figures, 7 tables, and 13 references.) (SLD)

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Three-dimensional Modeling in Linear Regression

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Paper presented at the annual meeting of the Southwest Educational Research Association, Dallas, January 27-29, 2000.

Abstract

Linear regression examines the relationship between one or more independent (predictor) variables and a dependent variable. By using the formula $\hat{y} = a + b_1x_1 + b_2x_2 + e$, regression determines the weights needed to minimize the error term (e) for a given set of predictors. With one predictor variable, the relationship between the predictor and the dependent variable is linear. With two predictors, this relationship becomes planar, and with three or more predictors, this relationship becomes hyperplanar. By examining 3-dimensional representations of the data, a researcher can gain greater insight into the data. The recent report of the APA Task Force on Statistical Inference, published in the August, 1999 issue of the American Psychologist, emphasized the value and importance of using graphics to understand and communicate data dynamics.

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Three-dimensional Modeling in Linear Regression

The influence of one or more factors on the outcome of a specific event or circumstance is an area of interest to many researchers. Extent of these influences can be investigated using linear regression. Linear regression can be used to explain the results of a study. An example would be if a committee responsible for the selection of incoming medical students tried to determine the influence of the previous grades, interview, and scores on the medical college aptitude test (MCAT) on the overall ranking of the candidates. Linear regression can also be used to predict. For example, if the same committee were interested in determining the how the ranking of the candidates would predict the performance of the incoming class in the first year of medical school, they could access the information from the previous class, along with their grades in their first year of medical school. By using regression, the committee could predict from the information from the previous class how the incoming class should perform.

Geneticist Francis Galton noted that the heights of sons had a tendency to regress toward the mean height of the population as compared to the height of their fathers. Sons of a tall man are more likely to be shorter, or closer to the mean height, while sons of a short man are more likely to be taller. Galton developed regression analysis in order to study this effect (Galton, 1889). Linear regression produces weights that are used on the predictor variables in order to produce optimal values for the dependent or outcome variable, minimizing error. The weight or regression coefficient is determined by dividing the sum of squares of the cross product by the sum of squares of the predictor variable (Hinkle, Wiersma, & Jurs, 1998: see also Christensen, 1996; Hocking, 1985). The formula used for simple linear regression is $\hat{y} = a + bx + e$, where $\hat{y}$ is the predicted outcome variable, "e" is the error term or $y - \hat{y}$, "a" is the y-intercept, and "b" is the regression coefficient (See Figure 1). This formula describes a regression line that has the

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smallest sum of squares for error (SSE). Both $\hat{y}$ and e are latent or synthetic variables, in that they are created as a result of regression, and may not bear any resemblance to the actual data. This formula can be converted into standard score form by setting "a" = 0 and $b = \beta(s_y/s_x)$. In standard score form, β is the regression coefficient or beta coefficient. Values for β can range from –1 to 1. With two predictor variables, the formula in raw score form is $\hat{y} = a + b_1x_1 + b_2x_2 + e$, and is $z_{\hat{y}} = \beta_1z_1 + \beta_2z_2$ in standard score form. Instead of being represented as a line, these data can be represented as a plane. Berk (1998) argued that plots of regression of two or more predictors in two dimensions do not provide comprehensive insight into the data set. In fact, Berk points out that three-dimensional plots can be used to represent multi-dimensional data, by "stacking" or layering additional variables along the x_2 axis.

In multiple regression, the "fit" of the plane to the data is often described by the multiple correlation coefficient, R, which can be determined using β. The multiple correlation coefficient is a Pearson product-moment correlation between the outcome variable, y, and the predicted outcome variable, $\hat{y}$. In the case of one predictor variable, R is equal to r_{xy}. In case of two or more uncorrelated predictor variables, R is equal to the sum of r_{x1y}, r_{x2y}, etc. By taking the square root of the sum of the products of each β and its corresponding correlation between x_i and y, R can be calculated (Hinkle, Wiersma & Jurs, 1998).

Another valuable tool for examining the relationship between predictor variables and the dependent variable are structure coefficients. These are zero-order correlations between the predictor variables and the synthetic variable divided by R (Pedhazur, 1982). Mathematically, this partitions the influence of each predictor variable on $\hat{y}$ as seen in the equation $r_S = r_{xy}/R$. As explained by Thompson and Borrello (1985), data may be misinterpreted if structure coefficients are not examined. Additionally, Thompson points out that examination of structure coefficients can provide information about the collinearity of the predictor variables and the presence of suppressor variables (Thompson, 1992).

This paper has two purposes: (a) to represent linear regression in a three-dimensional form to provide visual insight into the influences of various factors on ŷ, and (b) to investigate the value of including structure coefficients in the graphs. A data set found in Table 1 will be used periodically to generate the graphics. When values for specific coefficients are altered artificially in order to examine their effects, the data set will not be used and a dot plot will not be included in the graph. Graphs will be limited to two independent (predictor) variables and one dependent (outcome) variable. Graphs will not be standardized for better visualization of the data away from the x-, y-, and z-axes. More than two predictors and the value of multidimensional graphs will be investigated in a another study. Analysis was performed using SYSTAT 8.0. Graphs were generated by SigmaPlot 5.0. .

The use of computer graphics to explore and understand data is consistent with the recently-released report of the APA Task Force on Statistical Inference (Wilkinson & The APA Task Force on Statistical Inference, 1999). As the Task Force emphasized,

> As soon as you have collected your data, before you compute <u>any</u> statistics, <u>look at your data</u>. Data screening is not data snooping. It is not an opportunity to discard data or change values to favor your hypotheses. However, if you assess hypotheses without examining your data, you risk publishing nonsense.... Graphical inspection of data offers an excellent possibility for detecting serious compromises to data integrity. The reason is simple: Graphics broadcast; statistics narrowcast. (p. 597, emphasis in original)

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Certainly such admonitions are not new (Tukey, 1977). And several resources are available for researchers seeking guidance on the use of graphics to explore and understand data (cf. Chambers, Cleveland, Kleiner & Tukey, 1983; Cleveland, 1995; Wilkinson, 1999). But microcomputer software has been improved markedly in recent years as regards graphic capabilities, and so now these applications are readily available to applied researchers.

Linear Regression Using Two Predictor Variables

General Graphic Presentation

Figure 2 attempts to clarify the direction of the influence of each predictor on the outcome by holding one predictor constant while altering the other. Initially, b_1 is sequentially changed from –1 to 1, while b_2 is held at 0.5. In Figure 3, b_1 is held at 0.5 while b_2 is sequentially changed from –1 to 1. The scatterplot and values for the y-axis have been removed for the sake of clarity. Note in both examples, the plane rotates on the mean for x_1 (20) and x_2 (20), depending upon whether b_1 or b_2 is being altered. Additionally, at near-zero values for b_1 and b_2, the plane settles near the mean for y (20). This seems intuitive in that if x_1 and/or x_2 are useless predictors of y, then the best predictor for y is the mean of y (Thompson, 1992).

From this perspective, the first predictor variable changes the slope of the plane from left to right. Positive values for b_1 will result in a rise in the plane from left to right, while negative values result in left-to-right declination. In this perspective, the second predictor variable influences the slope from the front to the back. Positive values for b_2 will result in a rise in the plane from front to back, while negative values result in a declination of the plane in the same direction. Rotating the image in any direction can easily confuse this, as seen in Figure 4. In this figure, the regression plot is rotated 45° in each sequential frame. It is necessary to note the direction of the scale on each labeled axis when looking at graphics in three dimensions.

Using Uncorrelated Predictors

Figure 5 demonstrates the relationship between two predictors and one outcome using the equation $\hat{y} = 23.248 + -0.379x_1 + 0.217x_2 + e$ as derived from using Predictor 1 and Predictor 2 as seen in Figure 4. Table 2 shows the Pearson correlation matrix for all variables, while Table 3 contains the regression data for Predictor 1 and Predictor 2 versus Outcome. The value for b_1 is the same as in Figure 1, and the addition of the second set of values for Predictor 2 demonstrates the relationship between Predictor 2 and the outcome independent of the influence of Predictor 1 because x_1 is uncorrelated with x_2.

In this case, R^2 is equal to the sum of r_{x1y}^2 and r_{x2y}^2. Looking at the regression sums of squares (SOS_R) of this data, for Predictor 1 and the outcome variable, the SOS_R is equal to 2.736. The SOS_R for Predictor 2 is 0.895. Totaling these two values results in the same SOS_R obtained by regressing Predictor 1 and Predictor 2 against the outcome variable. Because the two predictor variables are uncorrelated, they do not share any of the sums of squares for $\hat{y}$. Therefore, any contribution of individual predictors toward explaining y is additive.

Using Correlated Predictors

In the previous data set, predictor variables were uncorrelated. However, correlation between predictor variables influences the graphic. Using Predictor 1 and Predictor 4, which has approximately the same correlation with y (-0.379 for x_1 and 0.290 for x_2), the regression equation becomes $\hat{y} = 22.936 + -0.314x_1 + 0.167x_2 + e$ with the correlation of x_1 with $x_2 = -0.393$ as seen in Figure 6. Note that the declination of the plane on the x_1 axis is lessened when the predictors are correlated. As each predictor variable only has a limited sums of squares that it can use to explain $\hat{y}$, any sharing of sums of squares through correlation with other predictor variables will limit its influence on $\hat{y}$. This is seen using the same comparison of SOS_R as losted in Table 4. The SOS_R for predictor 1 is 2.736 and SOS_R for Predictor 4 is 1.601. However, when Predictors 1 and 4 are regressed together with y, the SOS_R is 3.184, indicating that some of the sums of squares explained by the predictors overlap. The correlation between Predictor 1 and Predictor 4 is –0.393.

Regressing Predictor 3 and Predictor 5 with y provides insight into to the effects of positively correlated independent variables ($r = 0.421$, Figure 7). For Predictor 3 regressed with y, the SOS_R is 4.413, while the SOS_R for Predictor 5 with y is 3.666 (from Table 5). The SOS_R for both predictors with y is 5.703. Regardless of the sign of the correlation coefficient, collinearity reduces the usefulness of an individual predictor variable when using multiple predictors in most cases. Figure 8 demonstrates is an overlay of Figure 7 with the collinearity of the two predictors removed.

<u>Value of Structural Equations</u>

Because of the influence of collinearity of predictor variables on linear regression, proper interpretation of data requires more information (Thompson, 1992; Thompson & Borrello, 1985). In Figure 9, Predictor 4 is regressed with Predictor 6 against the outcome resulting in $b_1 = 0.664$, $b_2 = 0.592$ and $\underline{R}^2 = 0.296$ (adjusted $\underline{R}^2 = 0.213$). The Pearson correlation coefficients for these data sets are $r_{P4,y} = 0.290$, $r_{P6,y} = 0.174$, and $r_{P4,P6} = -0.630$. This is contradictory to the previous discussion in that the resulting $\underline{R}^2$ <u>increases</u> when these two correlated predictors are used. The SOS_R for Predictor 4 by itself is 1.601, while the SOS_R for Predictor 6 is 0.576. The SOS_R regressing both predictors is 5.619 as seen in Table 6. Figure 10 show an overlay of Figure 9 with the red grid representing the weights for Predictors 4 and 6 with the correlation removed. This is due to the presence of a suppressor variable. By definition, a suppressor variable is a variable that increases $\underline{R}^2$ with its addition and is not or weakly correlated with the dependent variable Thompson, 1992). In this case, Predictor 6 is modestly correlated with y ($r = 0.174$) while Predictor 4 is more correlated ($r = 0.290$). By examining the structure coefficients (r_S) of these two variables, it can be determined which explains the most of ŷ. The structure coefficients can be calculated by dividing $r_{x\hat{y}}$ by R. The correlation between Predictor 4 and ŷ ($r_{P4,\hat{y}}$) is 0.534, while the correlation between Predictor 6 and ŷ ($r_{P6,\hat{y}}$) is 0.320. The structure coefficient for Predictor 4 (r_{SP4}) is 0.9816, while r_{SP6} is 0.5882. Predictor 4 occupies 98% of the sums of squares explained and Predictor 6 occupies 59%. See Figure 11 for the addition of the r_S for this data set. Another method by which the presence of suppressor variable can be determined is by regressing the predictors with and without the suspected suppressor. Regressing Predictors 4 and 5 with and without Predictor 6 yields an R without Predictor 6 of 0.491, and an R with Predictor 6 of 0.564 as seen in Table 7. The r for Predictor 4 increased from 0.223 with out Predictor 6 to 0.540 with Predictor 6. The correlation coefficient for Predictor 5 decreased from 0.402 to 0.191 when Predictor 6 was added. All three predictors are correlated, as seen in Table 1.

Linear regression when used as a tool for prediction or explanation is usually indicative of extreme interest in the data. However, b weights, β weights and correlation coefficients offer only incomplete insight into the nature of the predictors. Viewing the data set graphically and including the "best fit" plane allows the investigator to see how much collinearity is affecting the data. In general, collinearity reduces the slope of the plane, due to sharing of the sums of squares by the predictor variables. However, when a suppressor variable is present, collinearity increases the slope of the plane. Normally, this would not be apparent unless the investigator chose to overlay the regression graph with a graph of the correlation coefficients. By adding the structure coefficients, the suppressor variable can be identified as it should be uncorrelated with the outcome variable, but be credited with having a positive effect on the prediction validity of the model. Three-dimensional representation of data sets helps further elucidate the data by helping the researcher visualize the data as well as the values of the various coefficients.

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Figure Caption

Figure 1. Simple linear regression with $b = -0.379$.

Figure 2. Linear regression with two predictor variables varying b_1 and holding b_2 constant at 0.5. In graph A, $b_1 = -1.0$ and $b_2 = 0.5$. In graph B, $b_1 = -0.75$ and $b_2 = 0.5$. In graph C, $b_1 = -0.5$ and $b_2 = 0.5$. In graph D, $b_1 = -0.25$ and $b_2 = 0.5$. In graph E, $b_1 = 0.0$ and $b_2 = 0.5$. In graph F, $b_1 = 0.25$ and $b_2 = 0.5$. In graph G, $b_1 = 0.5$ and $b_2 = 0.5$. In graph H, $b_1 = 0.75$ and $b_2 = 0.5$. In graph I, $b_1 = 1.0$ and $b_2 = 0.5$.

Figure 3. Linear regression with two predictor variables holding b_1 constant at 0.5 and varying b_2. In graph A, $b_1 = 0.5$ and $b_2 = -1.0$. In graph B, $b_1 = 0.5$ and $b_2 = -0.75$. In graph C, $b_1 = 0.5$ and $b_2 = -0.5$. In graph D, $b_1 = 0.5$ and $b_2 = -0.25$. In graph E, $b_1 = 0.5$ and $b_2 = 0.0$. In graph F, $b_1 = 0.5$ and $b_2 = 0.25$. In graph G, $b_1 = 0.5$ and $b_2 = 0.5$. In graph H, $b_1 = 0.5$ and $b_2 = 0.75$. In graph I, $b_1 = 0.5$ and $b_2 = 1.0$.

Figure 4. Linear regression with two predictor variables with sequential 45° rotation, starting at 0° and ending at 315°.

Figure 5. Linear regression with two predictor variables with $b_1 = -0.379$ and $b_2 = 0.217$.

Figure 6. Linear regression with two predictor variables with $b_1 = -0.314$ and $b_2 = 0.167$ (top). Side-by-side comparison of plots of previous predictors (bottom right) and current predictors (bottom left).

Figure 7. Linear regression with two positively correlated predictors with $b_1 = 0.361$ and $b_2 = 0.287$.

Figure 8. Linear regression with two positively correlated predictors with $b_1 = 0.361$ and $b_2 = 0.287$. Red grid represents linear regression with two positively correlated predictors with $b_1 = 0.482$ and $b_2 = 0.439$.

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Figure 9. Linear regression with $b_1 = 0.664$ and $b_2 = 0.592$ as derived from predictor variables 4 and 6.

Figure 10. Linear regression with $b_1 = 0.664$ and $b_2 = 0.592$ as derived from predictor variables 4 and 6. Red grid represents overlay of predictor variables 4 and 6 without collinearity, $b_1 = 0.290$ and $b_2 = 0.174$.

Figure 11. Linear regression with $b_1 = 0.664$ and $b_2 = 0.592$ as derived from predictor variables 4 and 6. Red grid represents overlay of predictor variables 4 and 6 without collinearity, $b_1 = 0.290$ and $b_2 = 0.174$. Blue grid represents an overlay of the structure coefficient for Predictor 4 and Predictor 6, with $b_1 = 0.9816$ and $b_2 = 05882$.

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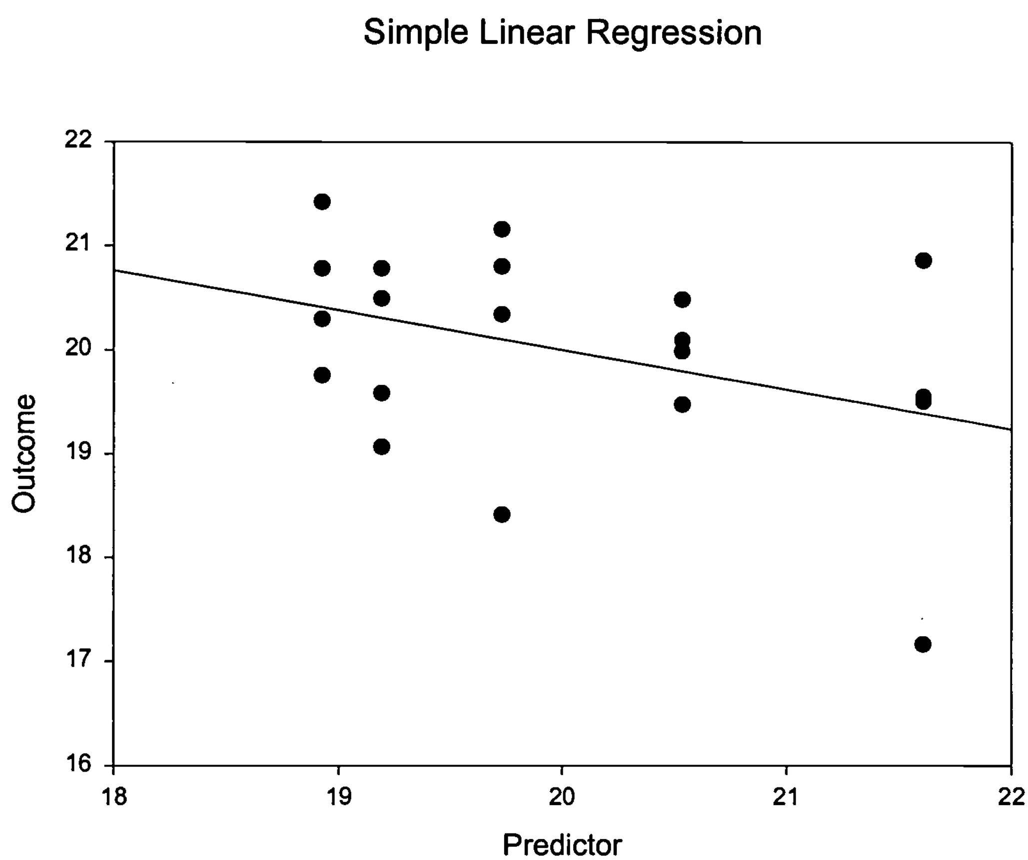
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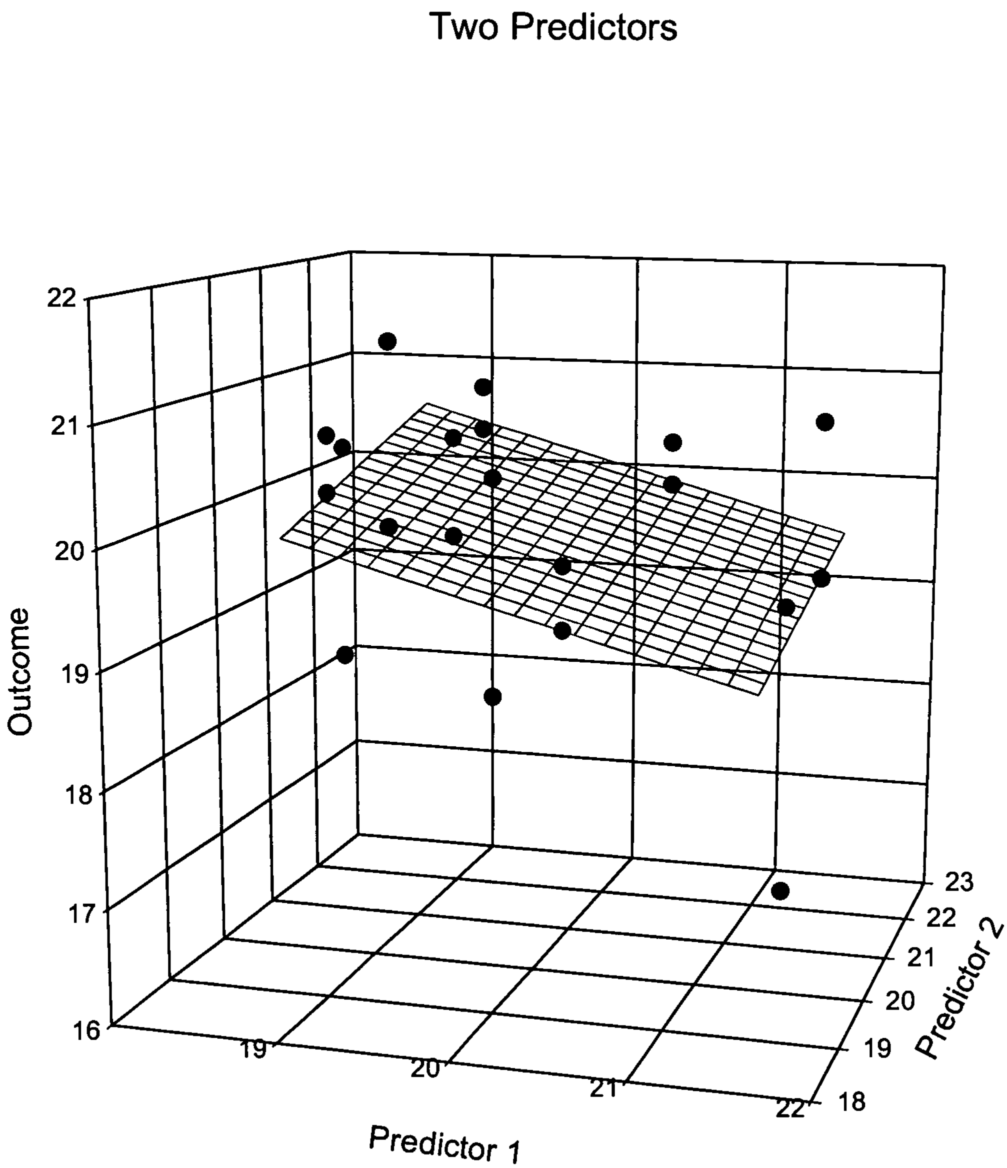
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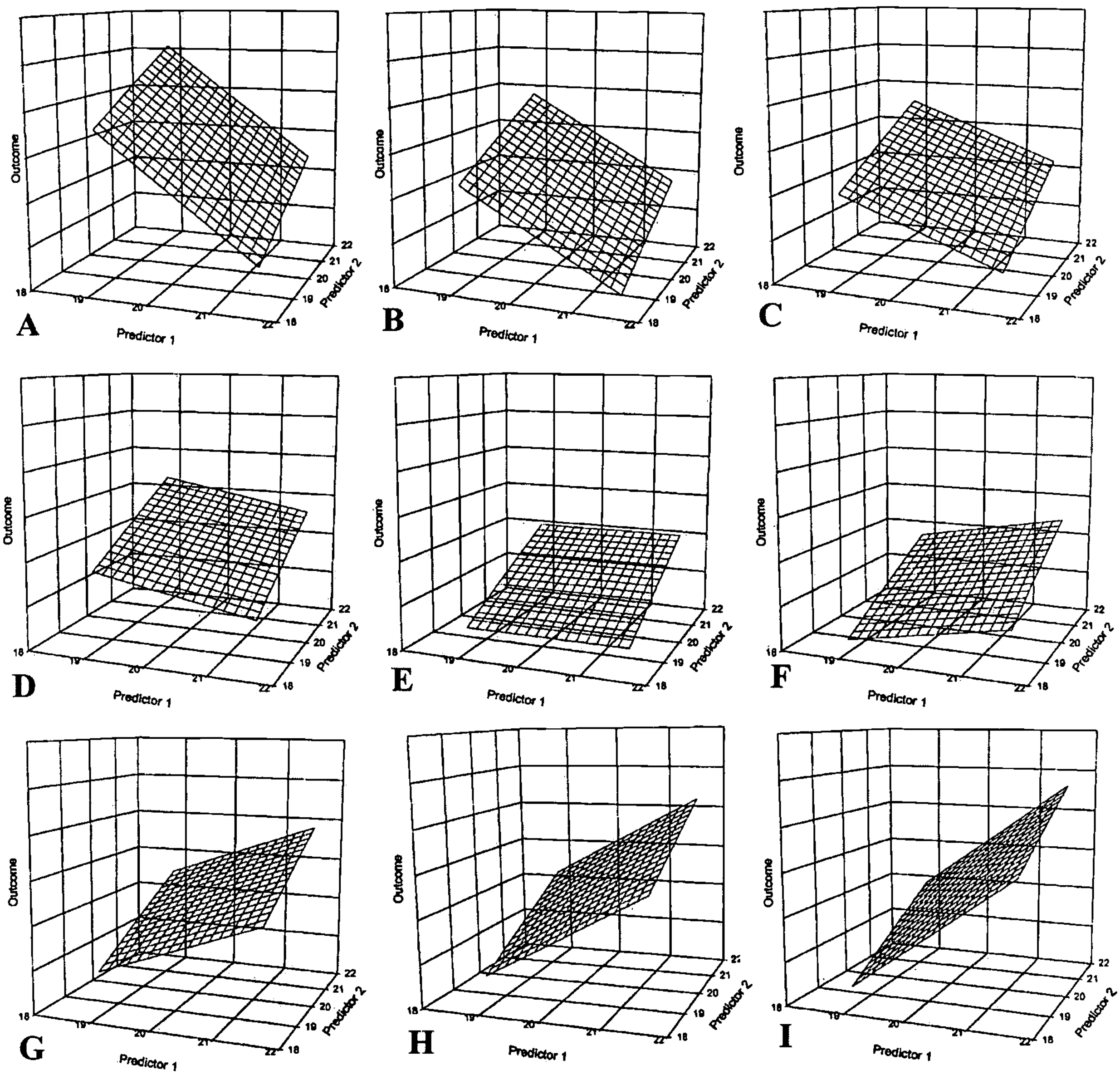
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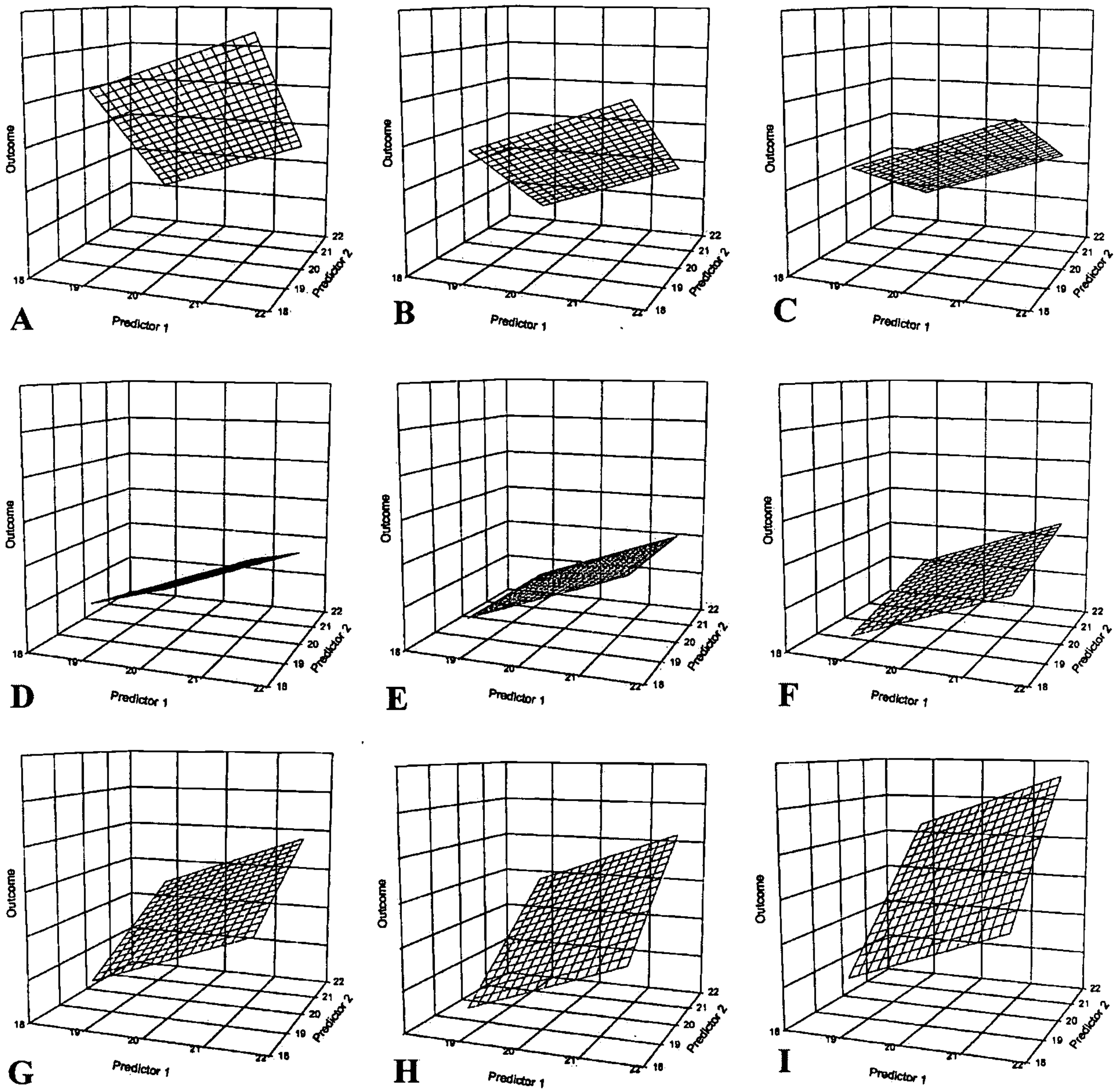

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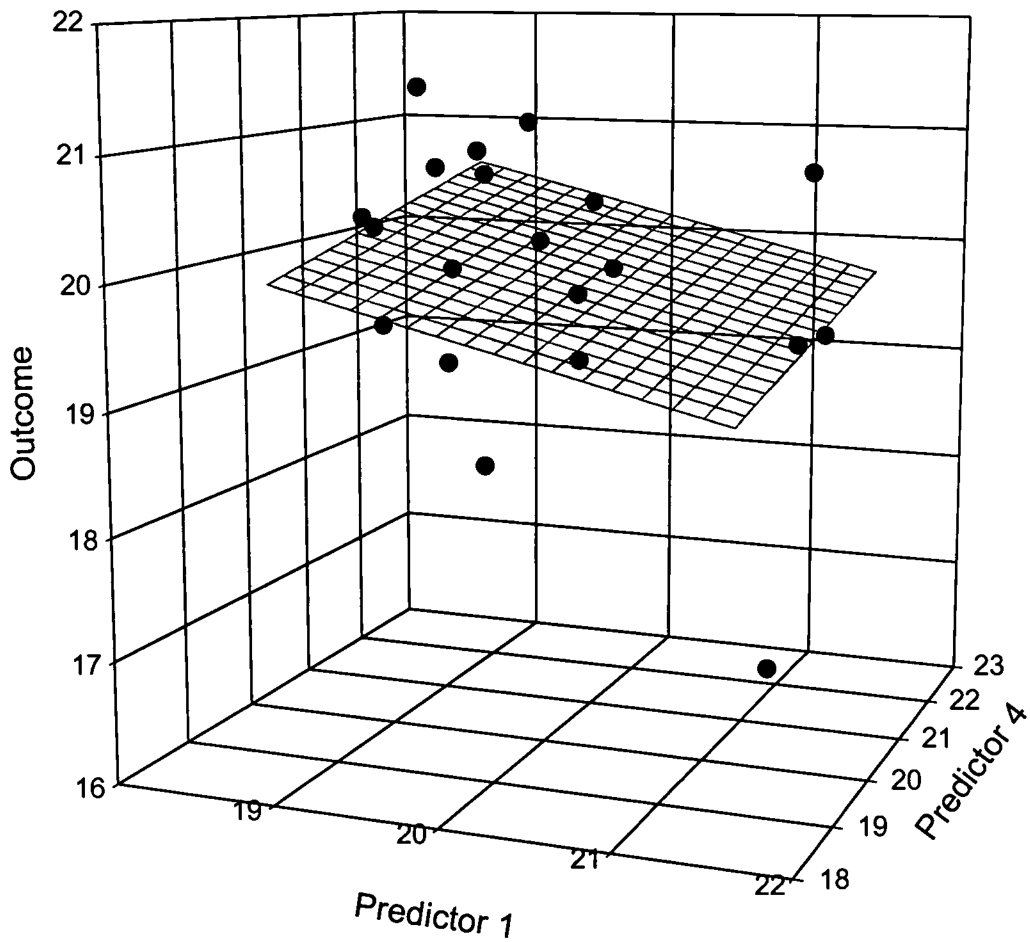
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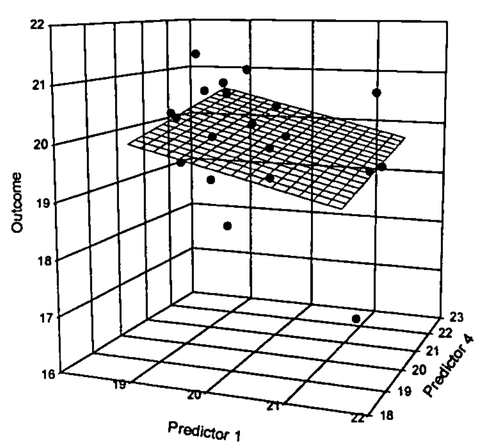
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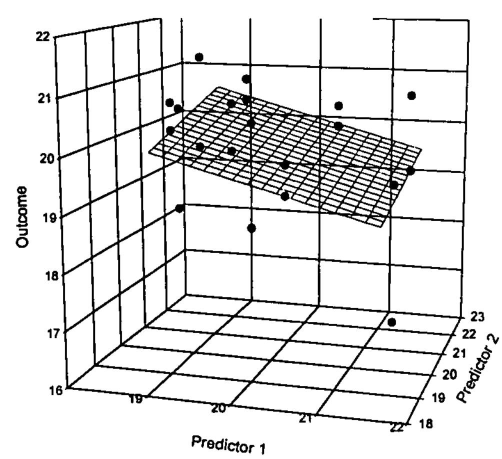
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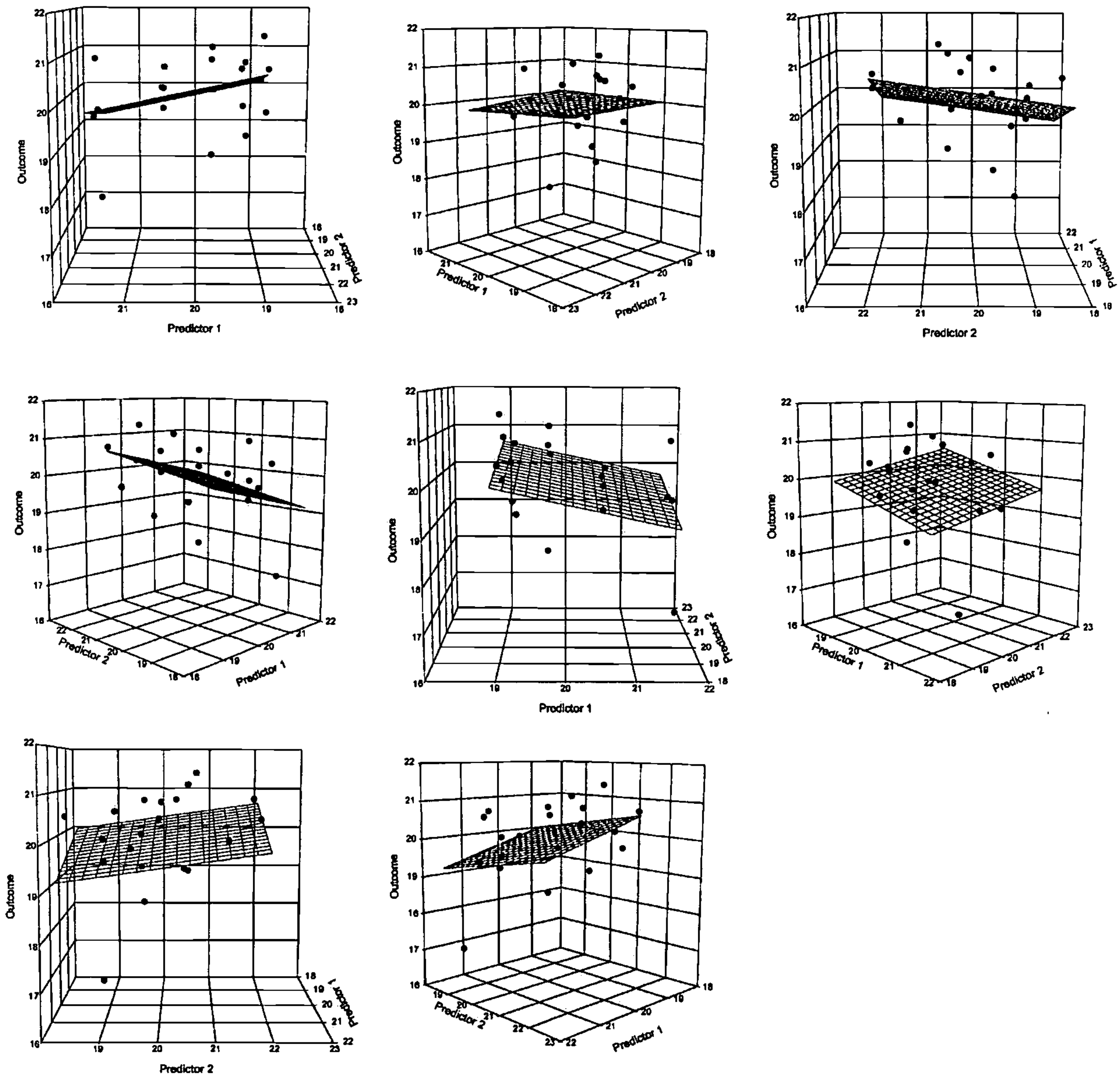
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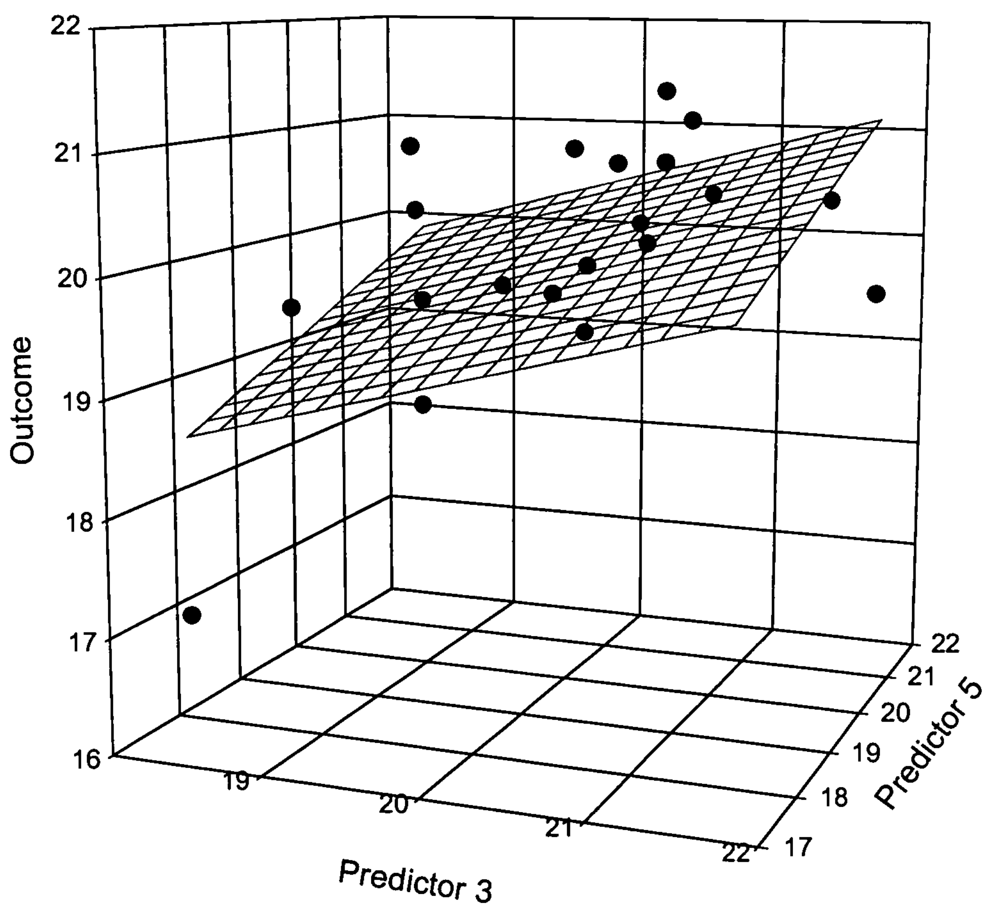
Positively Correlated Predictors



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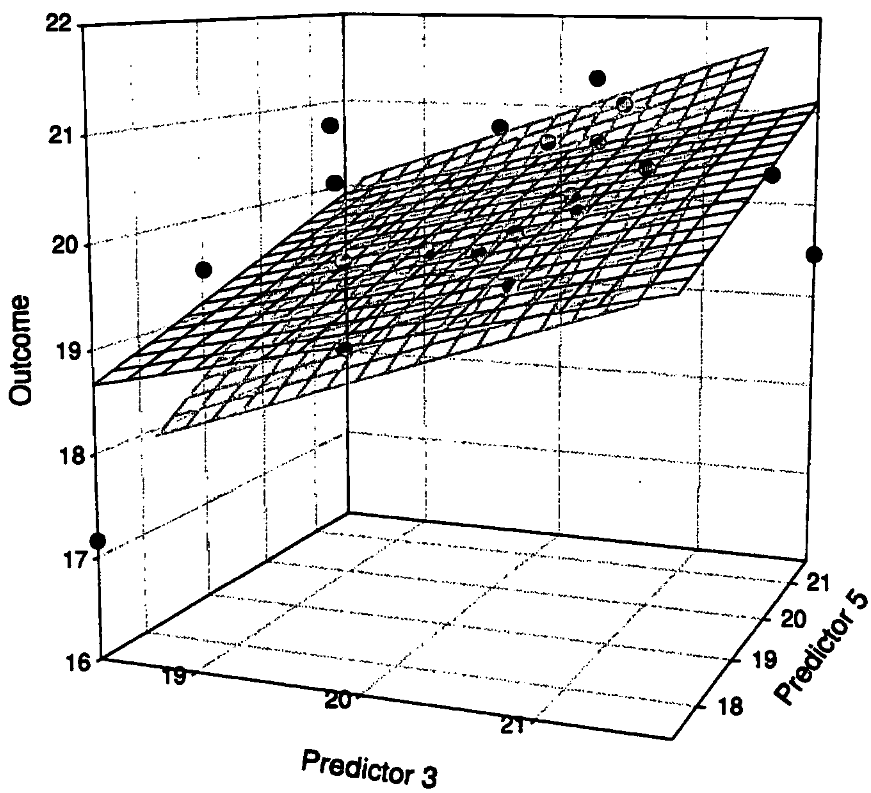
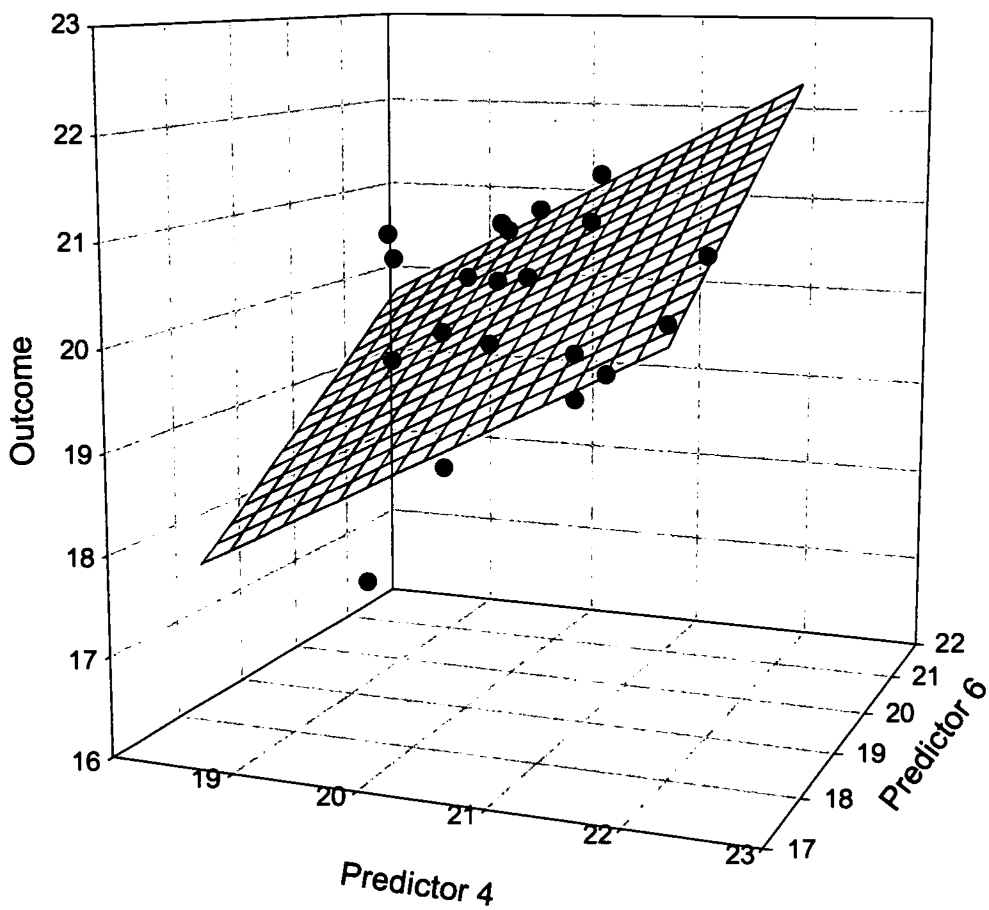
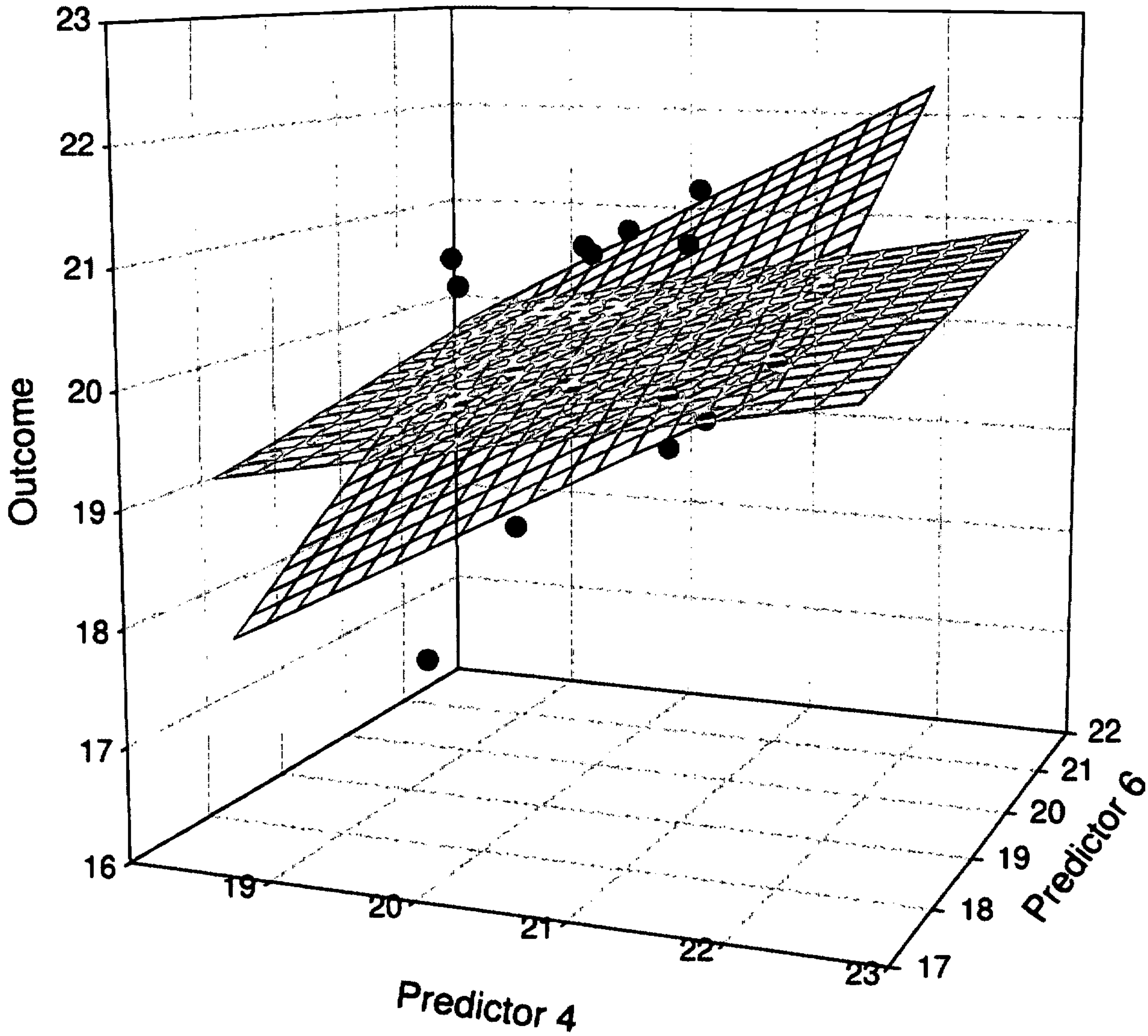


Linear Regression with Two Correlated Variables

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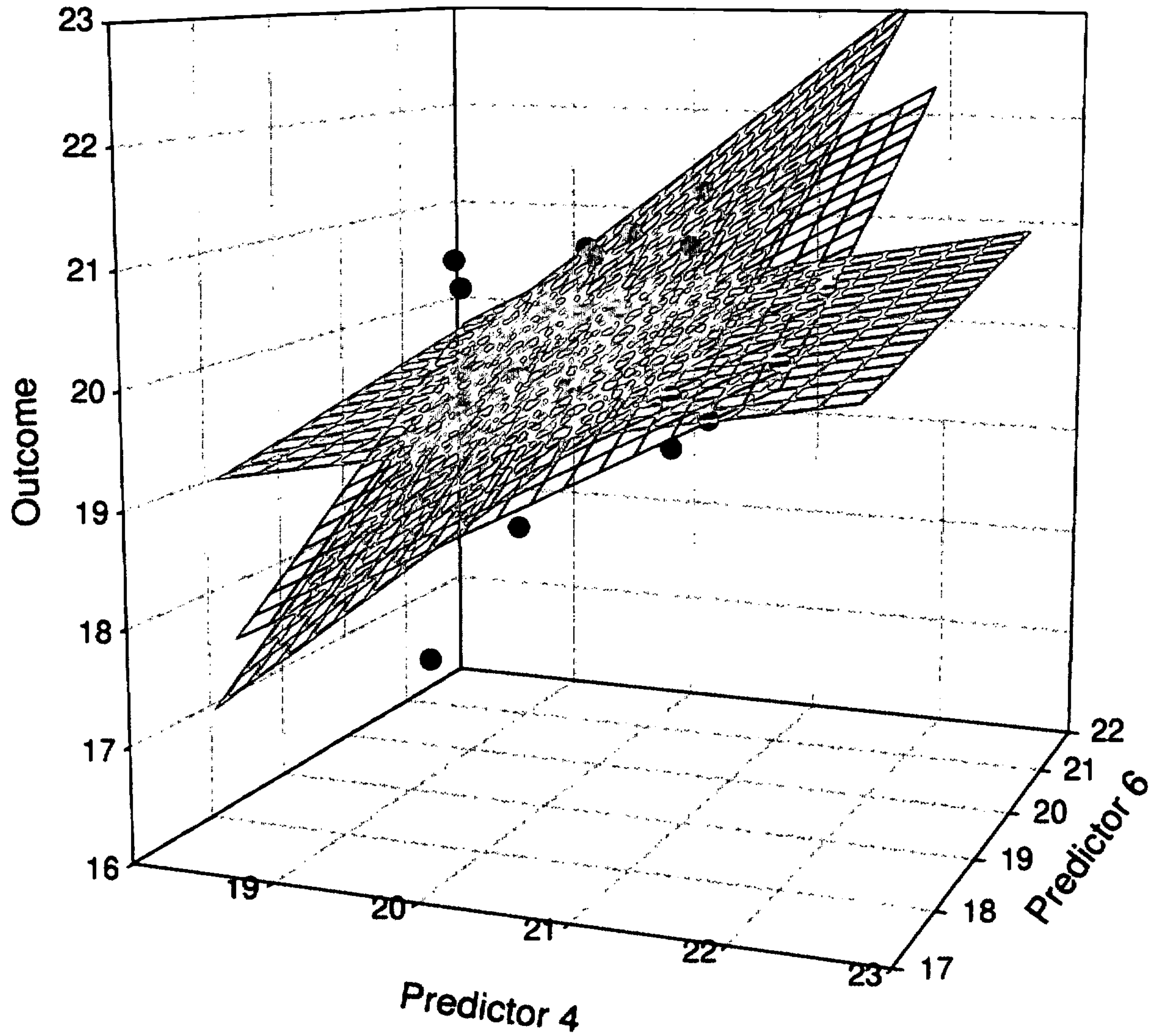
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Table 1

Data Set Used for Linear Regression

Outcome		Predictor 1	Predictor 2	Predictor 3	Predictor 4	Predictor 5	Predictor 6
19.553		21.61	19.338	19.162	19.718	19.488	20.24
20.094		20.537	21.545	20.576	19.64	19.925	21.286
20.799		19.732	19.89	20.386	19.662	19.889	20.641
20.778		19.195	18.786	19.646	20.297	21.399	21.116
20.296		18.927	19.338	20.579	19.924	19.732	19.904
21.42		18.927	20.662	20.598	20.704	20.303	20.223
19.582		19.195	21.214	18.595	19.35	18.549	19.095
20.345		19.732	20.11	19.087	21.979	19.566	18.004
19.988		20.537	18.455	20.386	18.923	19.148	21.652
20.86		21.61	20.662	20.806	20.068	19.481	19.781
19.753		18.927	20.662	19.768	21.384	19.325	18.4
20.491		19.195	21.214	21.681	19.026	20.357	19.841
18.415		19.732	20.11	18.873	19.657	20.294	19.378
19.474		20.537	18.455	21.746	18.945	21.679	20.997
19.506		21.61	20.662	19.755	20.467	20.51	20.224
17.166		21.61	19.338	18.393	19.058	17.365	19.21
20.48		20.537	21.545	20.857	18.217	20.556	21.488
21.158		19.732	19.89	20.76	20.537	20.344	19.275
19.067		19.195	18.786	19.834	20.541	21.022	20.03
20.778		18.927	19.338	18.512	21.904	21.07	19.216
Avg	20	20	20	20	20	20	20
sd	1	1	1	1	1	1	1

Table 2

Pearson correlation matrix

	Outcome	Predictor 1	Predictor 2	Predictor 3	Predictor 4	Predictor 5	Predictor 6
Outcome	1.000						
Predictor 1	-0.379	1.000					
Predictor 2	0.217	0.000	1.000				
Predictor 3	0.482	-0.005	0.125	1.000			
Predictor 4	0.290	-0.393	-0.080	-0.376	1.000		
Predictor 5	0.439	-0.312	-0.186	0.421	0.167	1.000	
Predictor 6	0.174	0.269	-0.144	0.509	-0.630	0.354	1.000

Table 3

Regression of Predictor 1 and Predictor 2

Dep Var: Outcome N: 20 Multiple R: 0.437 Squared multiple R: 0.191

Adjusted squared multiple R: 0.096 Standard error of estimate: 0.951

Effect	Coefficient	Std Error	Std Coef	Tolerance	t	P(2 Tail)
CONSTANT	23.248	6.174	0.000	.	3.765	0.002
Predictor 1	-0.379	0.218	-0.379	1.000	-1.739	0.100
Predictor 2	0.217	0.218	0.217	1.000	0.995	0.334

Analysis of Variance

Source	Sum-of-Squares	df	Mean-Square	F-ratio	P
Regression	3.631	2	1.815	2.007	0.165
Residual	15.373	17	0.904		

Table 4

Regression of Predictor 1 and Predictor 4

Dep Var: Outcome   N: 20   Multiple R: 0.409   Squared multiple R: 0.168

Adjusted squared multiple R: 0.070   Standard error of estimate: 0.965

Effect	Coefficient	Std Error	Std Coef	Tolerance	t	P(2 Tail)
CONSTANT	22.936	80.35	0.000	.	2.854	0.011
Predictor 1	-0.314	0.241	-0.314	0.846	-1.304	0.210
Predictor 4	0.167	0.241	0.167	0.845	0.694	0.497

Analysis of Variance

Source	Sum-of-Squares	df	Mean-Square	F-ratio	P
Regression	3.184	2	1.592	1.711	0.210
Residual	15.820	17	0.931		

Table 5

Regression of Predictor 3 and Predictor 5

Dep Var: Outcome   N: 20   Multiple R: 0.548   Squared multiple R: 0.300

Adjusted squared multiple R: 0.218   Standard error of estimate: 0.885

Effect	Coefficient	Std Error	Std Coef	Tolerance	t	P(2 Tail)
CONSTANT	7.037	4.819	0.000	.	1.460	0.162
Predictor 3	0.361	0.224	0.361	0.823	1.613	0.125
Predictor 5	0.287	0.224	0.287	0.823	1.284	0.216

Analysis of Variance

Source	Sum-of-Squares	df	Mean-Square	F-ratio	P
Regression	5.703	2	2.851	3.644	0.048
Residual	13.301	17	0.782		

Table 6

Regression of Predictor 4 and Predictor 6

Dep Var: Outcome N: 20 Multiple R: 0.544 Squared multiple R: 0.296

Adjusted squared multiple R: 0.213 Standard error of estimate: 0.887

Effect	Coefficient	Std Error	Std Coef	Tolerance	t	P(2 Tail)
CONSTANT	-5.114	9.470	0.000	.	-0.540	0.596
Predictor 4	0.664	0.262	0.663	0.603	2.531	0.022
Predictor 6	0.592	0.262	0.592	0.603	2.259	0.037

Analysis of Variance

Source	Sum-of-Squares	df	Mean-Square	F-ratio	P
Regression	5.619	2	2.809	3.568	0.051
Residual	13.385	17	0.787		

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Table 7

Regression of Predictor 4, Predictor 5 and Predictor 6

Dep Var: Outcome N: 20 Multiple R: 0.564 Squared multiple R: 0.318

Adjusted squared multiple R: 0.190 Standard error of estimate: 0.900

Effect	Coefficient	Std Error	Std Coef	Tolerance	t	P(2 Tail)
CONSTANT	-3.552	9.840	0.000	.	-0.361	0.723
Predictor 4	0.540	0.315	0.540	0.429	1.713	0.106
Predictor 5	0.191	0.262	0.191	0.622	0.729	0.477
Predictor 6	0.447	0.332	0.447	0.386	1.344	0.198

Analysis of Variance

Source	Sum-of-Squares	df	Mean-Square	F-ratio	P
Regression	6.049	3	2.016	2.490	0.097
Residual	12.955	16	0.810		

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